



# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

## B.A. DEGREE EXAMINATION – ECONOMICS

FOURTH SEMESTER – APRIL 2023

### UMT 4404 – ADVANCED MATHEMATICS FOR ECONOMICS

Date: 04-05-2023

Dept. No.

Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

#### PART – A.

(10 × 2 = 20 marks)

Answer ALL questions.

1. Write the first and second order conditions to find the maximum and minimum value of a function  $y = f(x)$ .
2. Define reorder costs & storage costs.
3. The utility a consumer derives from consuming the goods  $A$  and  $B$  can be assumed to be determined by the utility function  $U = 40A^{0.25}B^{0.5}$ . Find the marginal utility of  $A$  and  $B$ .
4. Write the Lagrange equation for constrained minimization with two variables.
5. What are the assumptions in the cobweb model?
6. Define Duopoly and Oligopoly market.
7. Write the formula that is used to find the final value of any variable growing continuously at a known annual rate from a given original value.
8. What is the interpretation of the definite solution in economics?
9. Write the solution for the homogeneous differential equation.
10. Find the particular solution to the differential equation  $\frac{dy}{dt} = 6y + 27$ .

#### PART – B

(5 × 8 = 40 marks)

Answer any FIVE questions.

11. Find the profit-maximizing output for a firm with the total cost function  $TC = 4 + 97q - 8.5q^2 + \frac{1}{3}q^3$  and the total revenue function  $TR = 58q - 0.5q^2$ .
12. A firm uses 200,000 units of a components in a year with demand evenly spread over the year. In addition to the purchase price, each order placed for a batch of components costs Rs 80. Each unit in stock over a year costs Rs 8. What is the optimum order size?
13. A firm faces the production function  $Q = 120L + 200K - L^2 - 2K^2$  for positive values of  $Q$ . It can buy  $L$  at Rs 5 a unit and  $K$  at Rs 8 a unit and has a budget of Rs 70. What is the maximum output it can produce?
14. Write the procedure to solve the cobweb difference equations and discuss about the stability and un-stability conditions.
15. Two duopolists  $X$  and  $Y$  react to each other's prices according to the functions
16.  $P_t^X = 240 + 0.9P_{t-1}^Y$ ,  $P_t^Y = 240 + 0.9P_{t-1}^X$ . If firm  $X$  sets an initial price of 2900, what will be its price be twenty time period latter? Assume that each firm adjusts price every alternate time period.

17. a) World reserves of minerals  $M$  are observed to have declined from 830 million tons to 657 million tons over the last 25 years. Assuming this decline has been continuous, calculate the annual rate of decline and then predict what reserves will be left in 10 years.
- b) Over the last 15 years a country's population has risen continuously at the same annual growth rate from 8.2 million to 11.9 million. What is this rate of growth?

For the differential equation  $\frac{dy}{dt} = 0.4y - 80$  with initial value  $y_0 = 180$ .

- a) derive the definite solution  
 b) use the solution to predict the value of  $y$  when  $t$  is 5.
18. Let the demand and supply function in a competitive market be  $Q_d = 35 - 0.5P$  and  $Q_s = -4 + 0.8P$ , and the rate of adjustment of price when the market is out of equilibrium be  $\frac{dP}{dt} = 0.25(Q_d - Q_s)$ . Derive and solve the differential equation to get a function for  $P$  in terms of  $t$  given that price is 37 in time period 0. Comment on the stability of the market.

### PART-C

(2 × 20 = 40 marks)

**Answer any TWO questions.**

19. a) A monopoly faces the total revenue schedule  $TR = 300q - 2q^2$  and the total cost schedule  $TC = 12q^3 - 44q^2 + 60q + 30$ . What is the profit maximizing output? (10 marks)
- b) Find whether any stationary point exist for the following functions for positive values of  $q$ , and say whether or not the stationary points are at the minimum values of the functions.
- i)  $MC = 30 + 0.4q^2$   
 ii)  $MC = 8.25q$  (10 marks)
20. a) A firm faces the production function  $Q = 6K^{0.4}L^{0.5}$ . Suppose it can buy input  $K$  at Rs 32 a unit and input  $L$  at Rs 8 a unit. Using substitution method find what combination of  $L$  and  $K$  should it use to maximize production if it is constrained by a fixed budget of Rs 36000?
- b) The utility a consumer derives from consuming the two goods  $A$  and  $B$  can be assumed to be determined by the utility function  $U = 40A^{0.25}B^{0.5}$ . If  $A$  costs Rs 4 a unit and  $B$  costs Rs 10 a unit and the consumer's income is Rs 600, using Lagrange's method find what combination of  $A$  and  $B$  will maximize utility?
21. a) In an agricultural market where the assumptions of the cobweb model apply, the demand and supply schedules are  $Q_t^d = 3450 - 6P_t$  and  $Q_t^s = -729 + 4.5P_{t-1}$ . Use difference equation to predict what price will be in the tenth period after an unexpected drop in quantity to 354, assuming that the market was previously in long-run equilibrium.

(14 marks)

b) Stocks of resource  $R$  are shrinking continually at an annual rate of 8.5 %. How much will remain in 30 years time if current stocks are 725000 units? (6 marks)

22. a) In a macroeconomic model  $C = 200 + 0.75Y$ ,  $E = C + I$  and  $I = 80$ ,  $\frac{dY}{dt} = 0.8(E - Y)$ , if  $Y_0 = 1200$ , derive a function for  $Y$  in terms of  $t$  and comment on the stability of this model.

b) Solve the differential equation  $\frac{dy}{dt} = 1.5y$  if the value of  $y$  is 34 when  $t = 0$ . Use the solution to predict the value of  $y$  when  $t = 7$ .

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